

## Organizational stuff

- 21<sup>st</sup> March: Andy Clark lecture in Amsterdam
- 23<sup>rd</sup> March: PP Seminar on the same topic! (enactivism, embodied and embedded PP)
- Birds of a feather report / announcements
- Call for new topics / discussions starting April 13<sup>nd</sup>

### Recall: Predictive Processing

**Brain as prediction machine**

- The brain continuously makes predictions about future sensory evidence based on its current best model of the causes of such evidence

**Bayesian Brain**

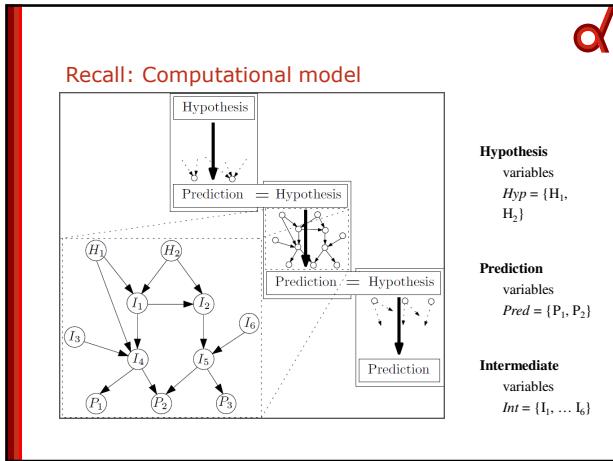
- The brain combines prior knowledge with sensory evidence (from various sources) in a Bayesian way

**Hierarchical Brain**

- The brain is organized in a hierarchical way, where "high level" information influences "low level" information and vice versa

## Recall: Key sub-processes

- Making **predictions** of expected input based on the generative models
- Comparing predicted inputs with actual inputs and **computing prediction error**
- **Explaining away** prediction errors (minimizing prediction error)
- **Learning** and adapting generative models



## Hyperprior

- Hyperpriors are **priors over priors**
- Define a distribution  $P(\text{Outcome})$  over Heads and Tails
- A hyperprior now describes a **distribution over  $x$** , such that  $P(\text{Outcome} = \text{Heads}) = x$  [and  $P(\text{Outcome} = \text{Tails}) = 1 - x$ ]
- What does it mean and what does it look like?

### Beta distribution



Probability density function of the Beta distribution for different parameter pairs:

- Beta(1, 1): Cyan curve, centered at 0.5
- Beta(2, 1): Green curve, centered at 0.33
- Beta(3, 1): Red curve, centered at 0.25
- Beta(4, 1): Blue curve, centered at 0.2

$$P(p; \alpha, \beta) = \frac{p^{\alpha-1} (1-p)^{\beta-1}}{B(\alpha, \beta)}$$

$$B(x, y) = \int_0^1 t^{x-1} (1-t)^{y-1} dt = \frac{\Gamma(x) \Gamma(y)}{\Gamma(x+y)}$$

### Beta function as hyperprior

- Hyperpriors are **conjugate priors** over the corresponding likelihood function
- Conjugate here means: if the likelihood distribution is of the family X, choosing a conjugate prior ensures that the **posterior distribution** is also of the family X
- In particular, a beta distribution is a conjugate prior over a binomial distribution (in this case: outcomes of coin tosses)
- Dirichlet distributions are conjugate priors over categorical distributions, Gaussians are conjugate priors over themselves, etc.

### Bayesian Updating



Bayesian updating of the Beta distribution over trials:

$s = \text{heads}, n-s = \text{tails}$

$$= \frac{x^{s+\alpha\text{Prior}-1} (1-x)^{n-s+\beta\text{Prior}-1}}{\int_0^1 (x^{s+\alpha\text{Prior}-1} (1-x)^{n-s+\beta\text{Prior}-1}) dx}$$

### Precision-weighted prediction error

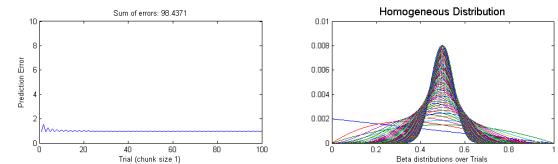
- Precision-weighted prediction error describes the **size of the effect** of a prediction error on the updating of the model
- The higher this **pwpe**, the **bigger the effect** on the generative model a prediction error is
- The higher this **pwpe**, the **more reducible uncertainty** there is in the environment
- We define this **pwpe** as the **KL divergence** between the hyperprior 'before' and 'after' updating with the new data
- Note that this is a **descriptive** measure, not a normative!



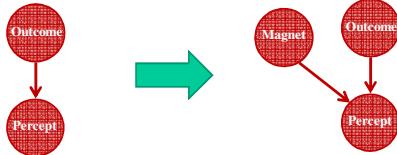
### Model updating cannot be the whole story

- There is evidence that 6mo infants process prediction errors, but the generative models undergo substantial post-natal development (Emerson et al., 2015)
- Immaturity of these models leads to sub-optimal processing of sensory stimuli in infants (Lee et al., 2012)
- How can models grow, incorporate new experiences, become more fine-grained etc.?
- There are computational arguments that Model Updating isn't the whole story, as well as empirical evidence!

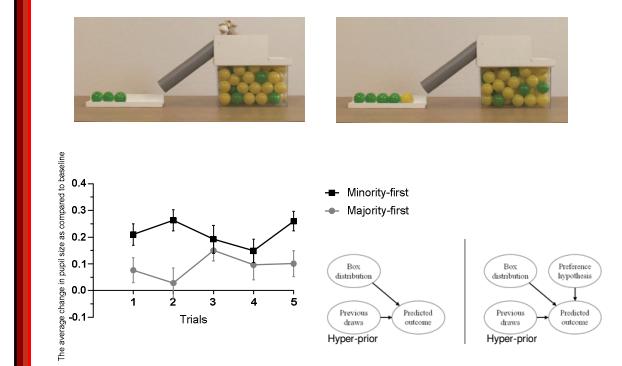
### Model Updating?



### Model Revision



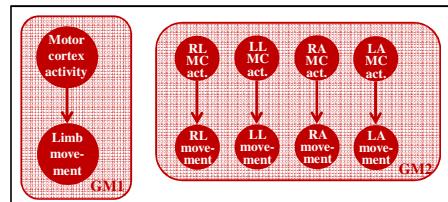
### Model Revision



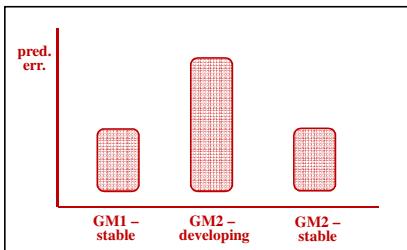
### Model Refinement

- Apart from adding context dependencies, adding hypotheses, changing structure of the models etc., another process that kicks in is Model Refinement
- In new & uncertain situations, very detailed predictions will carry lots of information but are likely to be false
- When you become more confident it becomes attractive to increase the *level of detail* of your predictions to make more informative predictions
- Think motor babbling in infants

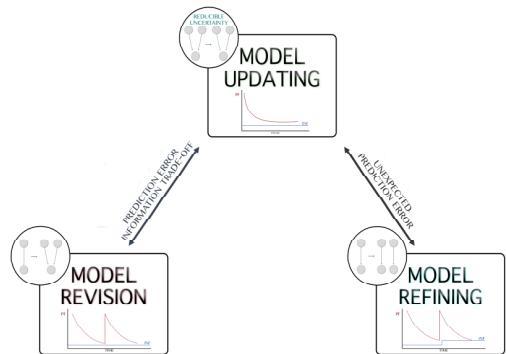
### Model Refinement



### Model Refinement



### Model Updating, Revision, and Refinement



### Model development open questions

- Where do the new hypotheses **come from**?
- How are they **integrated** with existing models?
- How are models '**split**' into more detailed models?
- How is the **trade-off** between informativeness and prediction error resolved? Is this context dependent? Governed by **hyperpriors**?
- How can we test this all?